KEK-TH-980

Decoupling Supersymmetry/Higgs without fine-tuning

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We propose a simple superpotential for the Higgs doublets, where the electroweak symmetry is broken at the supersymmetric level. We show that, for a class of supersymmetry breaking scenarios, the electroweak scale can be stable even though the supersymmetry breaking scale is much higher than it. Therefore, all the superpartners and the Higgs bosons can be decoupled from the electroweak scale, nevertheless no fine-tuning is needed. We present a concrete model, as an existence proof of such a model, which generates the superpotential dynamically. According to supersymmetry breaking scenarios to be concerned, various phenomenological applications of our model are possible. For example, based on our model, the recently proposed "split supersymmetry" scenario can be realized without fine-tuning. If the electroweak scale supersymmetry breaking is taken into account, our model provides a similar structure to the recently proposed "fat Higgs" model and the upper bound on the lightest Higgs boson mass can be relaxed.

arXiv:hep-ph/0409113v2 13 Sep 2004

Although the standard model (with a simple extention to incorporate neutrino masses and mixings) is in good agreement with almost of all the current experimental data, the model suffers from many problems. The gauge hierarchy problem is the most serious one. Since, in quantum theories, the Higgs boson mass is quadratically sensitive to new physics scale, an extreme fine-tuning is inevitable in order to obtain the correct electroweak scale if new physics scale lies at, for example, the Planck scale. In other words, the vacuum in the standard model is not stable against quantum corrections.

It is well known that this fine-tuning problem can be solved by introducing supersymmetry (SUSY) [1]. In SUSY models, there is no quadratic divergence of quantum corrections for the Higgs mass by virtue of supersymmetry, and hence the stability of the electroweak scale is ensured. However, none of superpartners have been observed yet, the SUSY must be broken at low energies. Once the SUSY is broken, the quadratic sensitivity returns and the Higgs boson mass receives quantum corrections of the SUSY breaking scale. Therefore, the SUSY breaking scale should be of the electroweak scale or smaller. Otherwise, the fine-tuning problem returns and the motivation of introduction of SUSY may fade away. Unfortunately, the lower bound on masses of superpartners and the lightest Higgs boson in the minimal supersymmetric standard model (MSSM) is being raised by the current experiments. In the MSSM, we have already faced the so-called "little hierarchy problem" and a fine-tuning at the accuracy of about 1 % in the Higgs potential is needed to obtain the correct scale of the electroweak symmetry breaking. This fact may indicate that we should admit the fine-tuning at this (or higher) level or the Higgs sector in the MSSM should be extended so that the lightest Higgs boson can be heavier [2] or the nature is fine-tuned intrinsically and SUSY is nothing to do with the gauge hierarchy problem [3].

Here let us reconsider the fine-tuning problem. The quadratic ultraviolet sensitivity of quantum corrections for the scalar mass is inevitable in field theories without SUSY or with broken SUSY. However, note that the following conditions are satisfied, the problem can be avoided: the Higgs boson is heavy enough to neglect the quadratic quantum corrections, but develops the electroweak scale VEV. Normally such a situation cannot be realized. This is because a usual Higgs potential in the standard model contains only one mass parameter such that

$$V(\phi) = \mu\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \tag{1}$$

where ϕ is the Higgs doublet, and $\mu < 0$ is the negative mass squared. The Higgs boson mass ($\sqrt{2|\mu|}$) is found to be the same order of or smaller than the scale of its VEV ($|\langle\phi\rangle| = \sqrt{|\mu|/2\lambda}$), unless the dimensionless coupling λ is taken to be large beyond the perturbative regions.

If the Higgs potential includes two (or more) hierarchical mass parameters, it may be possible to obtain a Higgs VEV much smaller than the Higgs boson mass itself through a combination among mass parameters. In neutrino physics, there is a famous example, namely the see-saw mechanism [4], which can effectively lead to the tiny neutrino mass scale through a relation among hierarchical mass scales. If a similar mechanism works in a Higgs potential the fine-tuning problem may be solved (the same arguments are seen in Ref. [5]). In this paper, we introduce a supersymmetric model which can realize such a situation.

We propose a simple Higgs superpotential with two mass parameters such that

$$W = mH_uH_d + \alpha^{-1}\frac{M^{3+2\alpha}}{(H_uH_d)^\alpha} \tag{2}$$

where H_u and H_d are the up and the down-type Higgs doublets, respectively, m and M are mass parameters,

and $\alpha > 0$ is a positive number. The first term is a mass term, while the second term is the so-called runaway superpotential. Suppose that the hierarchy $M \ll m$ in the following. Before discussing the origin of the superpotential, let us first show consequences derived from it.

The Higgs doublets obtain their VEVs through SUSY vacuum conditions and the electroweak symmetry is broken. The F-flat condition leads to

$$\langle H_u^0 H_d^0 \rangle = M^2 \left(\frac{M}{m} \right)^{\frac{1}{\alpha+1}} \tag{3}$$

where H_u^0 and H_d^0 are the electric charge neutral components in each Higgs doublets, and furthermore $\langle H_u^0 \rangle = \langle H_d^0 \rangle$ is required by the D-flat conditions. Note that since $M \ll m$ we can obtain the electroweak scale much smaller than M (and m) through the similar relation to the see-saw mechanism. Analyzing the superpotential and the (supersymmetric) Higgs potential including the D-term potentials, we can check the supersymmetric mass spectrum consistent with the supersymmetric Higgs mechanism, namely there are three would-be Nambu-Goldstone (NG) bosons, one real and one complex Higgs bosons and their superpartners, which play a role of component fields in the massive vector multiplets of Z and W bosons. Note that, in addition to them, there exists one (neutral) chiral Higgs multiplet with heavy mass $2(\alpha+1)m$. This heavy Higgs boson parameterizes the direction perpendicular to the F-flat direction of Eq. (3), on the other hand, the F-flat direction itself is bounded by only the D-flat conditions and is parameterized by the light Higgs bosons being the scalar components in the massive vector multiplets.

In order for the model to be realistic, SUSY should be broken. After the SUSY is broken, the soft SUSY breaking terms are taken into account in the Higgs potential. Since the direction perpendicular to the F-flat condition is bounded by the large mass scale m, Eq. (3) is approximately satisfied at a potential minimum, namely $\langle H_u^0 \rangle \simeq v^2 / \langle H_d^0 \rangle$ where $v^2 = M^2 (M/m)^{\frac{1}{\alpha+1}}$, even in the presence of any soft SUSY breaking terms smaller than m. On the other hand, $\langle H_u^0 \rangle$ itself is in general sensitive to the soft SUSY breaking terms larger than the electroweak scale, since the F-flat direction is bounded by only the D-flat conditions. If $\langle H_u^0 \rangle$ remains in the electroweak scale, in other words, $\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$ remains to be of order one even though the soft SUSY breaking terms are much higher than the electroweak scale, the fine-tuning problem can be solved.

Now we analyze the Higgs potential including the soft SUSY breaking terms. We parameterize them such that

$$\begin{aligned} V_{\rm soft} &= m_u^2 |H_u|^2 + m_d^2 |H_d|^2 \\ &+ B_1 m H_u H_d + B_2 \frac{m v^{2(\alpha+1)}}{(H_u H_d)^\alpha}. \end{aligned} \tag{4}$$

Let us consider some typical cases. Case (i): $|B_1| \sim |B_2| \gg |m_u| \sim |m_d|$. This case is naturally realized in the anomaly mediated supersymmetry breaking (AMSB) scenario [6, 7] with the sequestering ansatz [6], where $|B_{1,2}| \sim m_{3/2}$ and $|m_{u,d}| \sim 0.01 m_{3/2}$ ($m_{3/2}$ is the gravitino mass). The mass terms can be neglected as a good approximation. Considering a symmetry under $H_u \leftrightarrow H_d$, we can easily solve the stationary conditions analytically and find a potential minimum at $\langle H_u^0 \rangle = \langle H_d^0 \rangle = v(1 + \mathcal{O}(m_{3/2}/m))$. Note that the VEV is slightly shifted from the value in the SUSY limit due to the SUSY breaking effect. At this vacuum, the light Higgs bosons in the SUSY limit obtain their masses of order $m_{3/2}$ through the soft SUSY breaking terms. Furthermore, their superpartner Higgsinos also obtain masses of order $m_{3/2}$. This can be understood as follows. In the SUSY limit, there is no μ term (μ parameter in the chargino and neutralino mass matrices), since $\mu = 0$ is obtained under the F-flat condition. After SUSY is broken, the Higgs VEV is sifted by $v \times \mathcal{O}(m_{3/2}/m)$ from the value in the SUSY limit, and thus the F-flat condition is no longer exactly satisfied. As a result, the μ term of order $m_{3/2}$ is generated. This is the same mechanism discussed in [8]. Therefore, the electroweak scale can be stable almost independently of the soft SUSY breaking terms. We can raise the SUSY breaking scale without fine-tuning so that all the superpartners and also Higgs bosons are decoupled. Case (ii): $|m_u| \sim |m_d| \gg |B_1| \sim |B_2|$. This case is naturally realized in the gauge mediated SUSY breaking scenario [9]. We can neglect B-terms as a good approximation. As far as $|m_{u,d}| \ll m$, the F-flat condition of Eq. (3) is almost satisfied, and the Higgs potential can be approximately reduced into the form,

$$V \sim m_u^2 |H_u|^2 + m_d^2 \frac{v^4}{|H_u|^2} + D\text{-terms}. \tag{5}$$

As discussed above, the F-flat direction is bounded by only the D-term potential. Note that this situation is the same as for squarks and sleptons in the MSSM. In order to avoid squark and/or charged slepton condensations, which break the gauge symmetries of QCD and/or QED, soft mass squared for squarks and sleptons should be positive. If this is the case for the Higgs doublets, the electroweak scale can be stable as follows. For large soft masses, we can neglect the D-term potentials in Eq. (5) and find a potential minimum at $\langle H_u^0 \rangle \simeq v \sqrt{m_d/m_u}$ and then $\tan\beta \simeq m_d/m_u$. Therefore if two soft SUSY breaking masses are of the same order, the electroweak scale is stable even if the soft masses are very heavy. As in the case (i), through numerical analysis, we can find that the light Higgs bosons and Higgsinos obtain masses of order $m_{u,d}$. Again, all the superpartners and also Higgs bosons can be decoupled without fine-tuning. On the other hand, if m_u^2 and/or m_d^2 are negative the Higgs VEVs are found to be of order $|m_{u,d}|^2$ as in the MSSM. Thus the electroweak scale SUSY breaking is necessary and the superpartners cannot be decoupled. However, in

this case, we find that our model has a similar structure to the recently proposed "fat Higgs" model [2]. The electroweak symmetry is broken at the SUSY level, and the tree level upper bound on the lightest Higgs boson mass $\leq M_Z$ in the MSSM can be relaxed. In fact, through numerical analysis, we can find that the lightest Higgs boson mass can be 130 GeV, for example, even at the tree level. This phenomenology is worth investigating. Case (iii): $|m_{u,d}| \simeq |B_{1,2}|$. This case is realized in normal supergravity scenario. If both of the soft mass squareds are positive, we can obtain almost the same results in case (ii). In other cases, results are depend on input values of the soft SUSY breaking terms. More elaborate numerical studies are needed. We leave this issue for future works.

Finally we introduce a model which can naturally realize the superpotential of Eq. (2) as an effective superpotential. We present this model as an existence proof of a concrete model rather than a proposal of a specific model. One may be able to construct a simpler model than that we will present. The most important part in the superpotential is the second term, the runaway type superpotential. Although it is hard to belive that any perturbative theories can derive it, such a superpotential in fact can be generated dynamically in SUSY gauge theories [10], where M stands for the dynamical scale of a strong gauge interaction. As will be seen in the following, the runaway term can be generated in a composite model of the Higgs doublets. However in general it would not be essential for a model to be a composite model. Whatever a model is, a quit complicated dynamical model or a model inspired by string theories through some complicated (non-perturbative) structures etc., our aim can be accomplished if only the superpotential of Eq. (2) is finally generated as an effective superpotential. We may define the $U(1)_R$ charge for H_u and H_d as $-1/\alpha$ and for the mass term m as $+2(1+1/\alpha)$. According to the method in SUSY gauge theories developed by Seiberg and co-workers [11], there is a possibility that the superpotential of Eq. (2) can be effectively generated after integrating out other fields in a model, since the superpotential is the unique one consistent with the global $U(1)_R$ symmetry.

Now we present a model in five dimensions with the warped compactification of the fifth dimension on S^1/Z_2 [12] with a metric, $ds^2 = e^{-2r_c k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 dy^2$, where r_c and y is the radius and the angle of S^1, and k is the AdS curvature scale. We follow the formalism in Ref. [13]. The model is based on the gauge group $SU(N_c)_H \times SU(2)_L \times U(1)_Y$ with an integer $N_c \geq 3$. The particle contents are as follows:

	$SU(N_c)_H$	$SU(2)_L$	$U(1)_Y$
P	$\mathbf{N_c^*}$	$\mathbf{2}$	0
N_u	$\mathbf{N_c}$	$\mathbf{1}$	$+1/2$
N_d	$\mathbf{N_c}$	$\mathbf{1}$	$-1/2$
S	$\mathbf{1}$	$\mathbf{1}$	0
S_c	$\mathbf{1}$	$\mathbf{1}$	0
Z	$\mathbf{1}$	$\mathbf{1}$	0

Here $SU(N_c)_H$ is a strong gauge interaction newly introduced. Suppose that the singlet superfields (S and S_c) reside in the bulk with assigned Z_2-parity (even for S and odd for S_c), while the singlet (Z) and the preons (P, N_u and N_d) reside on the boundary branes at $y=0$ and $y=\pi$, respectively. The basic Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{bulk} &= \int d^4\theta r_c \omega(y)^2 \left(S^\dagger S + S_c^\dagger S_c\right) \\
&+ \int d^2\theta \omega(y)^3 S_c \left\{\partial_y - \left(\frac{3}{2}-c\right) r_c k \epsilon(y)\right\} S + h.c., \\
\mathcal{L}_{y=0} &= \int d^4\theta Z^\dagger Z + \int d^2\theta Z \left(\frac{S(0)^2}{M_5} - M_5^2\right) + h.c., \\
\mathcal{L}_{y=\pi} &= \int d^4\theta \omega(\pi)^2 K_{preons} \\
&+ \int d^2\theta \omega(\pi)^3 \frac{S(\pi)[PN_u][PN_d]}{M_5^{\frac{5}{2}}} + h.c., \qquad (6)
\end{aligned}$$

where M_5 is the five dimensional Planck scale, $\omega(y) = \exp(-r_c k|y|)$, c is the bulk mass for the bulk fields, $S(y)$ denotes the value of S at y, and K_{preons} denotes the Kahler potential for the preons. Normally only S (the Z_2 even field) can couple to the brane fields, and we have introduced the higher dimensional interaction terms naturally suppressed by the five dimensional Planck scale.

At low energies where $SU(N_c)_H$ becomes strong, the effective Lagrangian is expressed by the effective Higgs fields composed of the preons, $H_u \sim [PN_u]/\Lambda$ and $H_d \sim [PN_d]/\Lambda$ with the $SU(N_c)_H$ dynamical scale Λ, and new term in the superpotential,

$$W_{dyn} = (N_c - 2)\left(\frac{\Lambda^{3N_c-4}}{H_u H_d}\right)^{\frac{1}{N_c-2}}, \qquad (7)$$

is dynamically generated [10]. For simplicity, we assume the canonical Kahler potentials for the Higgs doublets in the following. Then the effective Lagrangian on the boundary brane at $y=\pi$ is rewritten as

$$\begin{aligned}
\mathcal{L}_{y=\pi} &= \int d^4\theta \omega(\pi)^2 \left(H_u^\dagger H_u + H_d^\dagger H_d\right) \qquad (8) \\
&+ \int d^2\theta \omega(\pi)^3 \left(\frac{\Lambda^2 S(\pi)(H_u H_d)}{M_5^{\frac{5}{2}}} + W_{dyn}\right) + h.c.,
\end{aligned}$$

In order to obtain the 4 dimensional effective Lagrangian, first solve the equation of motion for the bulk field S such as $\{\partial_y - (3/2-c) r_c k \epsilon(y)\} S = 0$, and find

the solution $S(x,y) = \tilde{S}(x) \times \exp[(3/2-c)r_c k|y|]$ with $\tilde{S}(x)$ being 4 dimensional superfield. Then substitute the solution into the above Lagrangian, and integrate it with respect to the fifth dimensional coordinate y. Furthermore, by rescaling and normalizing all the fields appropriately to make their Kahler potentials the canonical forms, we obtain a 4 dimensional effective superpotential,

$$\begin{aligned} W_{eff} &= Z\left(\frac{\tilde{S}^2}{N_S^2 M_5} - M_5^2\right) + \omega_c \left(\frac{\Lambda^2}{N_S M_5^{\frac{5}{2}}}\right)\tilde{S} H_u H_d \\ &+ (N_c-2)\frac{(\omega(\pi)\Lambda)^{\frac{3N_c-4}{N_c-2}}}{(H_u H_d)^{\frac{1}{N_c-2}}} \end{aligned} \quad (9)$$

where $\omega_c = \exp[(1/2-c)r_c k\pi]$ is defined, and $N_S^2 = [\omega_c^2 - 1]/(1-2c)k$ is the normalization factor of $\tilde{S}$. After integrating out $\tilde{S}$ and Z under the SUSY vacuum conditions, we finally obtain the effective superpotential of Eq. (2) with the identifications

$$\alpha = \frac{1}{N_c-2}, \;\; m = \omega_c \frac{\Lambda^2}{M_5}, \;\; M = \omega(\pi)\Lambda. \quad (10)$$

We can realize $M \ll m$ with a small warp factor $\omega(\pi) \ll 1$ and/or a large $\omega_c \gg 1$. Note that $M = \omega(\pi)\Lambda$ is the physical dynamical scale in 4 dimensional effective theory. Considering a condition $\Lambda \leq M_5$ for the $SU(N_c)_H$ gauge theory to be well-defined and Eq. (3), we can obtain the theoretical upper bound on M such as

$$M \leq v \times \left(\frac{M_5}{v}\omega_c\right)^{\frac{N_c-2}{3N_c-4}}. \quad (11)$$

Taking, for example, $N_c = 4$, $\omega_c \sim 1$ (or $c \sim 1/2$) and $M_5 \sim k \sim M_P \simeq 2.4 \times 10^{18}$ GeV, the reduced Planck mass, we find $M \leq 1500$ TeV and the warp factor $\omega(\pi) \sim 10^{-12}$.

Some comments are in order. Since our model is based on the results in SUSY gauge theories, the scale of the soft SUSY breaking terms should be smaller than the dynamical scale M for the consistency of the model. In order to incorporate large Yukawa couplings, our model would be extended. A simple way is to introduce a pair of elementary Higgs doublets and mass mixings (of order m) among them and the composite Higgs doublets. With the elementary Higgs doublets (large) Yukawa couplings can be written as in the MSSM, and fermion masses are generated once the elementary Higgs doublets develop their VEVs. We can show that the elementary Higgs bosons obtain the VEVs of the electroweak scale through the mass mixings, even though they are all heavy.

In summary, we have proposed a simple superpotential for the Higgs doublets, where the Higgs doublets develop their electroweak scale VEV at the SUSY level, while one Higgs chiral multiplet is very heavy. In a class of soft SUSY breaking terms, we have shown that the electroweak scale can be stable without fine-tuning so that all the sparticles and Higgs bosons can be decoupled from the low energy theories. We have presented a concrete model which can dynamically generate the superpotential. Various phenomenological applications of our model are possible according to soft SUSY breaking scales to be concerned. For example, in our model, we can realize the recently proposed "split supersymmetry" scenario [3] without fine-tuning (but there is no light Higgs boson). As mentioned above, if we consider the electroweak scale SUSY breaking, our model provides the similar structure to the recently proposed "fat Higgs" model. These applications are worth investigating.

We would like to thank Noriaki Kitazawa for useful discussions. N.O. would like to thank the Abdus Salam International Centre for Theoretical Physics, Trieste, during the completion of this work. This work is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

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